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Original article

Examining Indonesian Pre-Service Teachers' Beliefs on the Nature of Mathematics

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Abstract

Introduction. Beliefs about the nature of mathematics influence a teacher's mindset and, as a result, the way he or she teaches in the classroom. Several studies have been conducted in this area, but they have rarely focused on pre-service teachers. The implication is that the instruments used to measure these beliefs must be modified. This study aims to refine and validate a scale to measure pre-service teachers' beliefs about the nature of mathematics and to determine the demographic analysis results that influence these beliefs.

Materials and Methods. A scale development study was adopted to achieve the objectives of this study. The participants were 410 pre-service teachers from undergraduate programs at one University with A (excellent) accreditation in the capital city of Indonesia. We used factor analysis to obtain a valid and reliable instrument. We also used multiple regression analysis to look at the relationships between pre-service teachers' gender, academic major, academic level, and mathematical beliefs.

Results. This study established a valid and reliable scale that includes three factors that underlie beliefs about the nature of mathematics. One factor is related to the philosophy of traditional mathematics, namely objective, and the other two factors are related to the philosophy of constructivism mathematics, namely relevant and dynamic. Additionally, we discover that the impact of the academic major variable is more significant than the influence of the other variables (gender and academic level).

Discussion and Conclusion. Beliefs about the nature of mathematics are central to the professional development of mathematics teachers because these beliefs have an implicit impact or are related to the beliefs, views, conceptions, or attitudes of teachers about teaching and learning mathematics and, in turn, lead to choices and practices carried out in class. Therefore, the beliefs scale developed in our research will allow researchers and/or interested parties to know the extent to which teachers' subjective knowledge of mathematics is used to improve these beliefs and lead to more meaningful mathematics practices.

Keywords: nature of mathematics, philosophy of mathematics education, scale development, demographic analysis, regression models

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Оригинальная статья

Изучение представлений индонезийских учителей о природе математики

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Аннотация

Введение. Представления учителей о природе математики стали одной из фундаментальных переменных в рамках психологии математического образования, поскольку существенно влияют на учебную практику педагога в классе. Цель статьи – представить результаты исследования по изучению психометрической валидности и надежности представлений учителей о природе математики, определению результатов демографического анализа, влияющих на эти убеждения.

Материалы и методы. Для изучения проблемы исследования был проведен опрос, в котором приняли участие 410 преподавателей программ бакалавриата одного университета с аккредитацией А (отлично) в столице Индонезии. С целью получения действительного и надежного инструмента применялся факторный анализ. Для изучения взаимосвязей между полом учителей, академической специальностью, академическим уровнем и математическими убеждениями использовался множественный регрессионный анализ.

Результаты исследования. Представленная трехфакторная модель соответствует критериям валидности и надежности. Факторы, формирующие данную модель, отражают традиционную (объективную) и конструктивистскую (релевантную и динамическую) математику. В этом контексте актуальным фактором является взгляд на математические объекты как на неотъемлемую часть культуры и социальных интересов. Обнаружено, что влияние академической основной переменной является более значительным, чем других переменных – пола и академического уровня.

Обсуждение и заключение. Разработанная шкала убеждений позволит исследователям и/или заинтересованным сторонам определить, в какой степени субъективные знания учителей по математике используются для улучшения этих убеждений и ведут к более осмысленной математической практике.

Ключевые слова: природа математики, философия математического образования, масштабное развитие, демографический анализ, регрессионные модели

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Introduction

Mathematics-related beliefs have become one of the fundamental variables and become an interesting topic in the body of research on the psychology of mathematics education[1] [1]. This variable is fundamental because it becomes the basis for a person's attitudes and behaviors towards mathematics [2; 3] and significantly affects a teacher's instructional practices in the classroom[2] [4]. It is interesting because the messy construct of beliefs causes researchers to have different opinions about the position

[1] Thompson A.G. Teachers' Beliefs and Conceptions: A Synthesis of the Research. In: Grouws D.A. ed. Handbook of Research on Mathematics Teaching and Learning. New York: Macmillan Publishing Co, Inc; 1992. p. 127–146. Available at: http://psycnet.apa.org/psycinfo/1992-97586-007 (accessed 01.08.2022).

[2] Ibid.

of beliefs' whether it belongs to the cognitive or affective domain, or maybe both [5]. This classification is important to determine the measuring instrument used.

We found that most studies measure mathematics-related beliefs in three ways: using questionnaires [1; 6; 7], interviews [8–10] and some use picture analysis [11; 12] to identify young children's beliefs about mathematics. The first two analyses focus on measuring the attitude domain, while picture analysis seems closer to knowledge. We agree that beliefs are in the affective domain when expressed in the form of preferences. However, we also agree that beliefs belong to the cognitive domain when associated with knowledge. We defined beliefs as an individual's subjective knowledge of the degree of truth based on experience and expressed in a propositional attitude.

There are many dimensions of teachers' beliefs about mathematics education, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics [13]. However, research focused on teachers' beliefs about the nature of mathematics in the literature is scarce, even though beliefs about the nature have a role as a foundation for other beliefs[3] and have a key position in the professional knowledge of mathematics teachers[4]. As an implication, there are still few instruments to measure beliefs about the nature of mathematics.

Several studies assess beliefs about the nature of mathematics using questionnaires but mixed with other belief constructs, such as teaching and learning [14; 15]. Y. Purnomo has developed a scale to measure teacher beliefs about the nature of mathematics as an independent construct [1]. Purnomo's study resulted in two factors in beliefs about the nature of mathematics: relevant and dynamic factors, which are both related to constructivist mathematics. In addition to the limitations in measuring beliefs about traditional mathematics, it is also important how the scale is applied to a sample of pre-service teachers. Considering, they are in the golden period to build their knowledge and beliefs about how to work with mathematics and valuable practices for teaching mathematics. Therefore, this study examined the psychometric validity and reliability of pre-service teachers' beliefs about the nature of mathematics. We also identified how these beliefs were predicted by several variables, namely, gender, academic major, and academic level.

Literature Review

Beliefs about the nature of mathematics are a person's views, perceptions, or conceptions of mathematics as a whole as a discipline[5] [6]. Some researchers describe beliefs about the nature of mathematics using the category of philosophy of mathematics education [6; 16; 17]. Other categories use the epistemological views of mathematical knowledge [18; 19]. This study uses the category of philosophy of mathematics education, arguing that the development of mathematics as a discipline is closely related to its philosophy[6].

Chassapis presented numerous points regarding mathematics philosophy, which plays an essential role in professional mathematics teachers' knowledge[7]. The first argument states that the philosophy of mathematics and the fundamental characteristics of mathematics education are inextricably linked. The second point is that teachers' ideas, perspectives, conceptions, or attitudes about mathematics, teaching, and learning are implicitly influenced by or related to mathematical philosophy. The third argument rests on the unmistakable premise that mathematics philosophy is

[3] Perkkilä P. Primary School Teachers' Mathematics Beliefs and Teaching Practices. In: Mariotti M.A. ed. Proceedings of the Third Conference of the European Society for Research in Mathematics Education. Bellaria, Italy; 2003. p. 1–8.

[4] Chassapis D. Integrating the Philosophy of Mathematics in Teacher Training Courses: A Greek Case as an Example. In: Karen F., Bendegem J.P. Van eds. Philosophical Dimensions in Mathematics Education. New York: Springer; 2007. p. 61–79. Available at: http://users.uoa.gr/~dchasapis/papers/C_Philosophical%20Dimensions.pdf (accessed 01.08.2022).

[5] Thompson A.G. Teachers' Beliefs and Conceptions: A Synthesis of the Research; Perkkilä P. Primary School Teachers' Mathematics Beliefs and Teaching Practices.

[6] Chassapis D. Integrating the Philosophy of Mathematics in Teacher Training Courses: A Greek Case as an Example.

[7] Ibid.

inextricably linked to a thorough grasp of mathematics as subject knowledge to be taught.

In the philosophy of mathematics, there are three views that differ in how they approach mathematics as a scientific discipline: Platonism (including logicism), formalism, and intuitionism/constructivism. Based on the literature, two classifications may be drawn from some of these philosophical views: absolutism and fallibilism[8] [20]. One probable reason is that all philosophical views point to the categorization of mathematical objects/knowledge as static (absolutism) or dynamic (fallibilism)[9].

According to absolutists, mathematical truth is absolute; mathematics is one of, if not the only, domains of knowledge that is definite, unchanging, undeniable, and objective[10]. This viewpoint is comparable to Platonism (logicism) or formalism[11] [20; 21]. Absolutism is a philosophy that regards mathematics as a heavenly gift, a formal language free of errors and contradictions, waiting to be found and existing before human invention, and independent of human knowledge and a rigorous system of rules and procedures [5; 22; 23]. To put it another way, mathematical objects are commonly considered as true for use by their users. Furthermore, according to Ernest[12], absolutists regard mathematics as a separate science from human morals and values. In other words, mathematics is regarded as the sole science that can stand on its own.

Fallibilists claim that mathematical truth is not absolute and develops with time and necessity, in contrast to absolutists. Mathematics is a product of human invention that exists in the world of the human mind. Furthermore, fallibilists believe that mathematics is an inextricably linked aspect of human culture that cannot be isolated from physical knowledge and other disciplines[13]. In other words, fallibilism, humanism, and social constructivism are equivalent concepts[14].

The psychometric validity and reliability of pre-service teachers' beliefs about the nature of mathematics were examined in this study. We also examined how numerous characteristics, including gender, academic major, and academic level, predicted these beliefs.

Materials and Methods

The participants in this study were 410 pre-service teachers in two different departments, namely the department of elementary education and mathematics education at one University with an A (Excellent) accreditation in Jakarta. They are active students in the first year (52.9%) and final year (47.1%). They are 85.6% female and dominated by Javanese ethnicity. All respondents were informed of the purpose of the study and expressed their willingness (consent) to cooperate.

Starting from the work of Purnomo [1], which developed an instrument to measure beliefs about the nature of mathematics (BNM), we added items from several relevant references[15] [15; 16] and then reviewed the underlying factor structure. The initial scale for measuring beliefs about mathematics consists of 30 items using a 6-point Likert scale in the range of strongly disagree and strongly agree. This scale is written in Indonesian.

Item Pool of the Beliefs about Nature of Mathematics Scale

1. Mathematics is concerned with thought processes.
2. Mathematics is computation
3. Mathematics is a set of pre-existing and proper rules and procedures*.
4. Some mathematical principles and facts can be doubted and questioned*.

[8] Ernest P. The Philosophy of Mathematics Education. London: Routledge Falmer; 1991.

[9] Cooney T.J., Wilson P.S. On the Notion of Secondary Preservice Teachers' Ways of Knowing Mathematics. In: Owens D.T., Reed M.K., Millsaps G.M. eds. Proceedings of the Seventeenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education; 1995. p. 91–96. Available at: https://files.eric.ed.gov/fulltext/ED389594.pdf (accessed 01.08.2022); Dossey J.A. The Nature of Mathematics: Its Role and Its Influence. In: Grouws D.A., ed. Handbook of Research on Mathematics Teaching and Learning. New York: Macmillan Publishing Co, Inc; 1992. p. 39–48.

[10] Ernest P. The Philosophy of Mathematics Education; Hersh R. What Is Mathematics, Really? Oxford, New York: Oxford University Press; 1997.

[11] Ernest P. Social Constructivism as a Philosophy of Mathematics. Albany, New York: Suny Press; 1998.

[12] Ernest P. The Philosophy of Mathematics Education.

[13] Ibid.

[14] Hersh R. What Is Mathematics, Really?

[15] Ernest P. The Philosophy of Mathematics Education.

5. Mathematics is about the consistent arrangement of symbols*.
6. Mathematical principles, facts, and concepts are highly likely contradictory*.
7. Mathematics was discovered only by scientists.
8. Everyone can invent mathematics*.
9. The development of mathematics is closely related to other fields of science*.
10. Mathematics is a science that can stand alone.
11. Mathematics is an exact science*.
12. Mathematics is flawed*.
13. Mathematical truth is unquestionable*.
14. Mathematical truth is affected by time and human needs.
15. Mathematics is ensured to be in line with logic.
16. In mathematics, what is true can change*.
17. There are only two options in mathematics: correct or wrong.
18. Many people utilize mathematics in their daily lives*.
19. Because mathematics is abstract, it is difficult to apply in real life.
20. Mathematics comes from social needs.
21. Mathematics is a strict discipline.
22. Mathematics is ingrained into human culture*.
23. In mathematics, there is only one right solution.
24. Mathematics existed long before humans discovered it.
25. Mathematics was developed by humans, not by nature.
26. Mathematics is the study and application of symbols, rules, formulas, facts, and mathematical procedures.
27. Mathematical ideas exist in the human mind.
28. Mathematics is constructed in a structured and systematic manner.
29. What is learned in mathematics can be used in other fields*.
30. There are several approaches to answering mathematical problems appropriately*.

Note: * Items included in the 3-factor model.

First, we used factor analysis to obtain a valid and reliable instrument. We split into two groups of samples randomly. The first group for EFA consisted of 215 participants, including 87.9% female, 70.7% were from the primary education department, and 29.3% from the mathematics education department, and 54.9% were first academic year, and 45.1% is the last academic year. The second group for CFA consisted of 195 participants, including 83.1% female, 77.9% were from the elementary education department and 22.1% from the mathematics education department, 50.8% were first academic year and 49.2% is the last academic year.

There are 0.033% incomplete values, so the multiple imputation method is used to overcome incomplete data [1; 24]. The fifth iteration data was utilized to impute the data. We also performed convergent and discriminant validity, as well as internal consistency. In the next step, we also conducted multiple regression analysis to examine the associations between pre-service teachers' gender, academic major, academic level and pre-service teachers' mathematical beliefs. Data descriptions of each belief viewed from the demographic factor category were also analyzed to clarify the influence of demographic factors on each belief dimension.

Results

We used Horn's Parallel Analysis to determine the number of factors by comparing actual and simulated data. This method produces actual data more than simulative data and stops at the third factor. There are 22 of the 30 items used to describe the three factors. The twenty-two items had factor loadings in the range of 0.337 to 0.706. The values show that the factor loading of each item is significant because it is more than 0.32[16].

Based on the results of the EFA, we conducted a CFA with these three factors. The fit of the model for each measure exceeded the accepted criteria limit, namely NC = 1.470 with $p < 0.001$, RMSEA = 0.049, TLI = 0.911, CFI = 0.929, and SRMR = 0.064. This model retains three factors that cover 14 of the 22 EFA results items.

The results of CFA indicates that the fourteen items in the 3-factor BNM model have an adequate factor loading in the range of 0.40 to 0.75. This model also produces a relatively adequate CR for each factor, close to 0.7 (Table 1). Therefore, based on these two measures[17], we conclude that this model meets the acceptable criteria of convergent validity.

Table 1 also shows the results of the HTMT analysis, which demonstrates that the 3-factor model of this BNM meets the criteria for discriminant validity, which is less than 0.85 [25]. Table 1 also shows that each correlation in the factor measures a different construct because it is at a coefficient of less than 0.85. In other words, this model meets the criteria of discriminant validity [26].

[16] Tabachnick B.G., Fidell L.S. Using Multivariate Statistics. 6th ed. Boston, MA: Pearson Education, Inc; 2014.
[17] Malhotra N.K., Dash S. Marketing Research: An Applied Orientation. 6th ed. New Jersey: Pearson Education; 2011.

Table 1. **Construct validity and Internal consistency results for the BNM model**

Factor	M	SD	α	CR	AVE	Spearman Correlations (HTMT)		
						1	2	3
1. Objective	4.837	0.729	0.666	0.687	0.364	–	0.455** (0.225)	–0.162* (0.664)
2. Relevant	4.953	0.523	0.655	0.696	0.371	–	–	–0.088 (0.161)
3. Dynamic	3.611	0.902	0.649	0.662	0.252	–	–	–

Note: $^{**}p < 0.01$, $^{*}p < 0.05$.

Source: Hereinafter in this article all tables were made by the author.

The reliability coefficient of each factor can also be seen in Table 1. The reliability of each factor is relatively adequate, which is more than 0.6[18] [27], with a reliability coefficient of 0.666 for the objective factor, 0.655 for the relevant and 0.649 for the dynamic factor.

Demographics Analysis. The regression results of each demographic variable on each aspect of teachers' beliefs about the nature of mathematics are presented in Table 2. We also show each descriptive data of the beliefs component seen from the predictive variables in Table 3.

Table 2. **Multiple regression models for teachers' mathematical beliefs as predicted by each set of the independent variables**

Predictor	Beliefs' Factors								
	Objective			Relevant			Dynamic		
	B	SE B	β	B	SE B	β	B	SE B	β
Gender	–0.194	0.138	–0.101	–0.016	0.099	–0.011	0.044	0.172	0.018
Academic Major	0.297	0.129	0.170*	0.313	0.093	0.249***	–0.455	0.161	–0.210**
Academic Level	0.264	0.106	0.182*	0.096	0.076	0.092	–0.043	0.132	–0.024

Notes: $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$.

Table 3. **Descriptive data for beliefs variables and demographic factors**

	Gender		Academic Major		Academic Level	
	Female	Male	Elementary Education	Mathematics Education	First year	Last year
	(n = 162)	(n = 33)	(n = 152)	(n = 43)	(n = 99)	(n = 96)
Objective						
M	4.866	4.698	4.775	5.052	4.742	4.936
SD	0.702	0.846	0.744	0.635	0.811	0.620
Min	2.750	2.500	2.500	2.750	2.500	3.270
Max	6.000	5.750	6.000	6.000	6.000	6.000
Relevant						
M	4.9573	4.9297	4.886	5.186	4.931	4.975
SD	0.516	0.562	0.536	0.399	0.546	0.500
Min	3.500	3.500	3.500	4.500	3.500	3.500
Max	6.000	5.830	6.000	5.830	5.830	6.000
Dynamic						
M	3.600	3.667	3.705	3.285	3.591	3.633
SD	0.898	0.933	0.873	0.935	0.937	0.867
Min	1.000	2.000	1.000	1.750	1.500	1.000
Max	5.250	5.750	5.750	5.000	5.500	5.750

[18] Nunnally J.C., Bernstein I.H. Psychometric Theory. 3rd ed. New York, NY: McGraw-Hill; 1994.

As shown in Table 2, academic major variables have a significant effect on each factor on beliefs about the nature of mathematics, namely for objective with $\beta = 0.170$ and $p = 0.023$, for relevant with $\beta = 0.249$ and $p = 0.001$, for dynamic with $\beta = -0.210$ and $p = 0.005$. Based on Table 3, the mean obtained by the sample with mathematics education major was significantly higher than elementary education major for both the objectivism and relevant variables, while the mean obtained by the sample with elementary education major (M = 3.705, SD = 0.873) was significantly higher than mathematics education major (M = 3.285, SD = 0.935) for dynamic variables.

Discussion and Conclusion

The three-factor model produced by this study has met the criteria for validity and reliability. The three factors forming this model reflect traditional mathematics (objective) and constructivism mathematics (relevant and dynamic). The relevant factor in this context is the view of mathematical objects as an inseparable part of the culture and social interests. This view can be associated with social constructivism or fallibilism[19] [23]. This study's dynamic view is that mathematics is not an exact science, but dynamic towards social change and technological developments[20] [1]. Lastly, objective factor in this study can be associated with a static view, instrumentalism, absolutism, which views mathematical truth as absolute, perfect, and unquestionable[21]. In other words, mathematical objects are often taken for granted by their users and are seen as the only science that can stand.

This research also shows that mathematics majors are more likely than elementary education majors to see mathematics as human activity and culturally relevant. The elementary education major sample, on the other hand, is more dynamic than the mathematics education major sample when it comes to perceiving mathematics as a scientific field. Finally, the sample of mathematics study programs is more absolute in its approach to mathematics than the sample of elementary education majors. This research's findings are closely related to the study of mathematics in the mathematics education study program's curriculum, which is more particular in the abstract realm. It limits the possibility of studying the dynamic of the mathematical object of study. This differs slightly from the major of primary education curriculum, which involves more interaction with physical things in everyday life than the major of mathematical education.

The research findings also demonstrate that numerous beliefs between the objective and relevant belief dimensions are held by our sample, as evidenced by the correlations in Table 2, notably among participants with a mathematics education major (Table 3). They think that mathematics is a useful subject in everyday life, but that it is also an absolute entity that can only be accepted if it is true and without faults. This conclusion supports earlier research [5; 28; 29], such as Purnomo et al. (2016), which looked at a pre-service elementary school teacher who was undertaking fieldwork experience and found discrepancies between the beliefs and classroom practice. This research also discovered that ideas about the nature of mathematics have an impact on instructional practice and other aspects of beliefs. Purnomo (2017a) observed comparable results in another study, claiming that believing in one construct might lead to contradiction in beliefs and practice.

We believe that the findings of this study's suggestions are critical for the growth of teacher education, particularly for those who are still in college. Despite the fact that their views had been established from their first interactions with mathematics, the ideal moment for establishing and correcting beliefs that were important to practice based on the aims of mathematics education itself was during the college education period. The teacher education curriculum should not only focus on understanding mathematical content, but also on how that content is to be delivered, as well as the underlying philosophical underpinning. Practices those are relevant to the application of mathematics both in the framework of instruction in schools, and in other settings that make mathematical awareness a human

[19] Ernest P. The Philosophy of Mathematics Education.
[20] Hersh R. What Is Mathematics, Really?
[21] Ibid.

activity and integrated in culture and social life, should be included in the curriculum.

While the findings of this study are beneficial, we acknowledge that it has certain limitations. The findings have limited application to other groups since we only used pre-service teachers from the elementary education and mathematics education departments. Future studies should focus on how this idea may be applied to a more diverse group. These two departments were chosen because they are uniquely qualified to teach mathematics in elementary and secondary schools.

In addition to the purpose of evaluating instruments to measure beliefs about the nature of mathematics, this study also aims to analyze the demographic factors that influence these beliefs. This study established a valid and reliable scale that includes three factors that underlie beliefs about the nature of mathematics: an objective view, a relevant view, and a dynamic view. Absolutism and fallibilism are two facets of mathematics education philosophy that are reflected in all three. Objectivism in the context of this research includes and is related to traditional mathematical views, Platonism, instrumentalism, or absolutism. While relevant and dynamic, they are related to fallibilism, humanism, or social constructivism. Our findings also show that students majoring in mathematics education and those majoring in primary education have quite different views on the nature of mathematics, notably objective beliefs, which is more prevalent in the mathematics education major. The discrepancy between the dimensions of belief in this study's findings also provides an intriguing subject for future researchers to investigate the factors that impact it, particularly in terms of their mathematical knowledge, because the two are intertwined.

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